

DO NOW

Find the derivative of:

$$f(x) = (x^2 + x)^5$$

$$\begin{aligned} y &= u^5 & u &= x^2 + x \\ \frac{dy}{du} &= 5u^4 & \frac{du}{dx} &= 2x + 1 \\ \frac{dy}{du} &= 5(x^2 + x)^4 \\ \frac{dy}{dx} &= 5(x^2 + x)^4(2x + 1) \\ \frac{dy}{dx} &= (10x + 5)(x^2 + x)^4 \end{aligned}$$

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3.4 The Chain Rule - Day 2

Recall:

$$\frac{d}{dx}[f(g(x))] = \underbrace{f'(g(x))}_{\text{derivative of outer function}} \cdot \underbrace{g'(x)}_{\text{derivative of inner function}}$$

The General Power Rule:

If $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then:

$$\begin{aligned} \frac{dy}{dx} &= n[u(x)]^{n-1} \cdot u'(x) \\ \text{OR} \\ y' &= n u^{n-1} \cdot u' \end{aligned}$$

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Examples - without 'u' substitution:

$$\begin{aligned} 1. \quad y &= (2x - 5)^3 \\ y' &= 3(2x - 5)^2 \cdot 2 \\ y' &= 6(2x - 5)^2 \end{aligned}$$

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$$\begin{aligned} 2. \quad f(x) &= \sqrt{3 - 2x} \\ f(x) &= (3 - 2x)^{\frac{1}{2}} \\ f'(x) &= \frac{1}{2}(3 - 2x)^{-\frac{1}{2}} \cdot (-2) \\ f'(x) &= \frac{-1}{\sqrt{3 - 2x}} \end{aligned}$$

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$$\begin{aligned} 3. \quad y &= \frac{2}{3x + 1} \\ y &= 2(3x + 1)^{-1} \\ y' &= 2 \cdot (-1)(3x + 1)^{-2} (3) \\ y' &= \frac{-6}{(3x + 1)^2} \end{aligned}$$

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$$\begin{aligned} 4. \quad y &= \sqrt[3]{(x^2 - 1)^2} \\ y &= (x^2 - 1)^{\frac{2}{3}} \\ y' &= \frac{2}{3}(x^2 - 1)^{-\frac{1}{3}} (2x) \\ y' &= \frac{4x}{3\sqrt[3]{x^2 - 1}} \\ y' &= \frac{4x}{3(x^2 - 1)^{\frac{1}{3}}} \end{aligned}$$

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$$\begin{aligned}
 5. \ y &= \sqrt[3]{3x^2 - x + 1} \\
 y &= (3x^2 - x + 1)^{1/3} \\
 y' &= \frac{1}{3} (3x^2 - x + 1)^{-2/3} (6x - 1) \\
 y' &= \frac{6x - 1}{3(3x^2 - x + 1)^{2/3}} \\
 y' &= \frac{6x - 1}{3\sqrt[3]{(3x^2 - x + 1)^2}}
 \end{aligned}$$

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$$\begin{aligned}
 6. \ y &= \sqrt{\frac{2x+1}{x}} \\
 y &= \left(\frac{2x+1}{x}\right)^{1/2} \\
 y' &= \frac{1}{2} \left(\frac{2x+1}{x}\right)^{-1/2} \cdot \frac{d}{dx} \left[\frac{2x+1}{x}\right] \\
 y' &= \frac{1}{2} \left(\frac{2x+1}{x}\right)^{-1/2} \cdot \left[\frac{x(2) - (2x+1)(1)}{x^2} \right] \\
 y' &= \frac{1}{2} \left(\frac{x}{2x+1}\right)^{1/2} \cdot \left[\frac{x(2) - (2x+1)}{x^2} \right] \\
 y' &= \frac{1}{2} \left(\frac{x}{2x+1}\right)^{1/2} \cdot \left[\frac{2x - 2x - 1}{x^2} \right] \\
 y' &= \frac{-1}{2x^2} \left(\frac{x}{2x+1}\right)^{1/2} \\
 y' &= \frac{-\sqrt{x}}{2x^2 \sqrt{2x+1}} \qquad y' = \frac{-1}{2x^2} \sqrt{\frac{x}{2x+1}}
 \end{aligned}$$

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$$\begin{aligned}
 7. \ y &= x(3x - 9)^3 \\
 y' &= x \frac{d}{dx} [(3x - 9)^3] + (3x - 9)^3 \frac{d}{dx} [x] \\
 y' &= x \cdot 3(3x - 9)^2 (3) + (3x - 9)^3 (1) \\
 y' &= 9x(3x - 9)^2 + (3x - 9)^3 \\
 y' &= (3x - 9)^2 (9x + 3x - 9) \\
 y' &= (3x - 9)^2 (12x - 9)
 \end{aligned}$$

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